Profilometer

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1 Math

The geometry of the laser emitter-camera system can be fully described with three variables (laser plane to camera distance, camera altitude angle, and camera axial angle).

Introducing a flat mirror into the system adds three new degrees of freedom (camera-mirror distance, two orientation angles) if we assume the camera's field of view to be fully reflected by the mirror. Under this assumption any plane-camera-mirror system has an equivalent plane-camera system. Should this assumption not hold for the entire FoV, it must be artificially restricted. A partially reflected FoV is more difficult to work with as well as completely useless for our purposes.

Given two planes in 3-space, any point not in either of these planes defines a non-linear bijective map between them (fig. 1). In case of parallel planes there are no singularities. In case of intersecting planes there is one line in each plane which maps to infinity. Imposing an orthonormal grid on both planes such that the intersection line is the x axis and the origin is the point closest to the external point yields the following:

$$x' = \frac{d.x.sin(\alpha + \beta)}{d.sin(\alpha + \beta) - y.sin(\alpha)}$$

$$y' = \frac{d.y.sin(\beta)}{d.sin(\alpha + \beta) - y.sin(\alpha)}$$

One of these planes is the laser plane, which has a fixed position. The other is the camera-normal plane, which can be placed wherever. If we place it so that the normal to the laser plane at the origin contains the camera we get the following (fig. 2):

$$b = d.cos(\sigma)$$

$$x_c = 0$$

$$y_c = d.sin(\sigma)$$

$$\Delta_x = b.tan(\frac{\phi_x}{2})(\frac{2X + 1}{X_m} - 1)$$

$$\Delta_y = b.tan(\frac{\phi_y}{2})(\frac{2X+1}{X_m}+1)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_c \\ y_c \end{pmatrix} + R(\psi)\begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix}$$

PCM to PC conversion here.

$$e^{e^e}$$